

# On Control of Large-Scale Hyperbolic PDEs via Continuum Approximations<sup>★</sup>

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**Abstract:** This extended abstract considers PDE continua as approximations of large-scale hyperbolic PDEs. The continuum approximation aspect is considered from the viewpoints of approximating both solutions to hyperbolic PDEs and solutions to backstepping kernel equations, which can be employed in stabilization of such systems. Finally, we provide discussion on alternative ways of constructing continuum approximations, as well as on potential alternative frameworks for PDE continua.

*Keywords:* large-scale systems hyperbolic PDEs PDE continua backstepping

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## 1. INTRODUCTION

The notion of PDE continua was coined in Alleaume and Krstic (2025) under the framework of infinite (continuum) ensembles of hyperbolic PDEs, where the inspiration for the class of systems came from traffic, such as lane-free models as in Malekzadeh et al. (2021), or multi-class models with continuum classes like drivers' age. Thereafter, Humaloja and Bekiaris-Liberis (2025c, 2026a) established that such continuum hyperbolic PDEs can act as continuum approximations of large-scale hyperbolic PDEs, motivated by models of large-scale traffic networks in Herty and Klar (2003); Yu and Krstic (2021); Burkhardt et al. (2021), large-scale blood flow networks in Bikia (2021); Reymond et al. (2009), and large-scale epidemics spreading networks in Guan et al. (2020); Kitsos et al. (2022).

Stabilization of hyperbolic PDEs using backstepping has been actively studied for various classes of such systems. Since the seminal work of Di Meglio et al. (2013) for  $n+1$  hyperbolic systems, various classes of  $n+m$  hyperbolic systems have also been addressed by Hu et al. (2016); Auriol and Di Meglio (2016); Coron et al. (2017, 2021); Auriol and Bresch-Pietri (2022). The class of continuum hyperbolic PDEs considered in Alleaume and Krstic (2025) can be viewed as a continuum counterpart of  $n+1$  systems as  $n \rightarrow \infty$ , and hence, we refer to it as  $\infty+1$  hyperbolic system. The backstepping methodology has been since extended to more general classes of continuum PDEs in Humaloja and Bekiaris-Liberis (2026a,b), where  $\infty+m$  and  $\infty+\infty$  hyperbolic systems, respectively, were considered.

This extended abstract considers the recent developments of continuum approximations of large-scale hyperbolic systems. For the sake of conciseness, we focus on  $n+1$  and the

respective  $\infty+1$  hyperbolic systems from Humaloja and Bekiaris-Liberis (2025b,c), but the presented theory and results also extend to  $n+m$ ,  $\infty+m$ , and  $\infty+\infty$  hyperbolic systems; see Humaloja and Bekiaris-Liberis (2026a,b). Already in Alleaume and Krstic (2025), the  $\infty+1$  continuum ensemble was viewed as a continuum counterpart of the  $n+1$  finite ensemble, and the formal connection between these two systems was made in Humaloja and Bekiaris-Liberis (2025c). In particular, we showed in Humaloja and Bekiaris-Liberis (2025c) that, if the parameters of an  $n+1$  and an  $\infty+1$  system and the respective initial conditions are connected appropriately, the solution to the  $\infty+1$  system acts as approximation of the solution to the  $n+1$  system. Moreover, an analogous approximation result applies to the respective backstepping kernel equations, such that a stabilizing state-feedback law for an  $n+1$  system can be designed based on the respective  $\infty+1$  backstepping kernels. In Humaloja and Bekiaris-Liberis (2025b), we developed a systematic numerical method for solving  $\infty+1$  kernel equations based on the power series approach from Vazquez et al. (2023a,b); Lin et al. (2024). Comparison of solving the  $n+1$  and  $\infty+1$  kernels with power series showed that the continuum approximation approach of solving the  $\infty+1$  kernels instead of the exact  $n+1$  kernels gains in computational complexity when  $n$  is sufficiently large, particularly as the continuum is independent of  $n$ . Hence, the potential (and demonstrated) gain in computational complexity is the main motivation for studying continuum approximations of large-scale hyperbolic PDEs.

The rest of this extended abstract is organized as follows. In Section 2, we present the class of  $\infty+1$  hyperbolic systems and show how its solution can act as a continuum approximation of a respective (large-scale)  $n+1$  hyperbolic system. In Section 3, we apply the continuum approximation approach to PDE backstepping, and show that the  $\infty+1$  backstepping kernels can approximate the exact  $n+1$  kernels and provide a stabilizing control law for the  $n+1$  system. In Section 4 we discuss aspects related to construction of continuum approximations and comment on open problems.

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*Notation.* We use the standard notation  $L^2(\Omega; \mathbb{R})$  for real-valued Lebesgue integrable functions on a domain  $\Omega \in \mathbb{R}^d$  for some  $d \geq 1$ . For some  $n \in \mathbb{N}$ , we denote  $E = L^2([0, 1]; \mathbb{R}^{n+1})$ , equipped with the inner product

$$\langle (\mathbf{u}_1^1), (\mathbf{u}_2^2) \rangle_E = \int_0^1 \left( \frac{1}{n} \mathbf{u}_1^T(x) \mathbf{u}_2(x) + v_1(x) v_2(x) \right) dx, \quad (1)$$

and the respective continuum space is denoted by  $E_c = L^2([0, 1]; L^2([0, 1]; \mathbb{R}) \times \mathbb{R})$  and equipped with the inner product

$$\langle (\mathbf{u}_1^1), (\mathbf{u}_2^2) \rangle_{E_c} = \int_0^1 \left( \int_0^1 u_1(x, y) u_2(x, y) dy + v_1(x) v_2(x) \right) dx, \quad (2)$$

which can be viewed as a continuum limit of  $E$  as  $n \rightarrow \infty$ . Finally, we denote by  $\mathcal{T}$  the triangular set

$$\mathcal{T} = \{(x, \xi) \in [0, 1]^2 : \xi \leq x\}. \quad (3)$$

## 2. CONTINUUM APPROXIMATION OF LARGE-SCALE HYPERBOLIC PDES

The class of  $\infty + 1$  hyperbolic systems is of the form

$$u_t(t, x, y) + \lambda(x, y) u_x(t, x, y) = \int_0^1 \sigma(x, y, \zeta) u(t, x, \zeta) d\zeta + W(x, y) v(t, x), \quad (4a)$$

$$v_t(t, x) - \mu(x) v_x(t, x) = \int_0^1 \theta(x, \zeta) u(t, x, \zeta) d\zeta, \quad (4b)$$

with boundary conditions

$$u(t, 0, y) = Q(y) v(t, 0), \quad v(t, 1) = U(t). \quad (5)$$

The aim of this section is to find such parameters  $\lambda, \mu, \sigma, W, \theta, Q$  of (4), (5) that the solution  $(u, v)$  would approximate (in some appropriate sense) the solution  $(\mathbf{u}, v)$  of a respective (large-scale)  $n + 1$  hyperbolic system of the form

$$\mathbf{u}_t(t, x) + \mathbf{\Lambda}(x) \mathbf{u}_x(t, x) = \frac{1}{n} \mathbf{\Sigma}(x) \mathbf{u}(t, x) + \mathbf{W}(x) v(t, x), \quad (6a)$$

$$v_t(t, x) - \mu(x) v_x(t, x) = \frac{1}{n} \mathbf{\Theta}(x) \mathbf{u}(t, x), \quad (6b)$$

with boundary conditions

$$\mathbf{u}(t, 0) = \mathbf{Q} v(t, 0), \quad v(t, 1) = U(t), \quad (7)$$

where the parameters are such that

$$\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_n) \in C^1([0, 1]; \mathbb{R}^{n \times n}), \quad (8a)$$

$$\mu \in C^1([0, 1]; \mathbb{R}), \quad (8b)$$

$$\mathbf{\Sigma} = (\sigma_{i,j})_{i,j=1}^n \in C([0, 1]; \mathbb{R}^{n \times n}), \quad (8c)$$

$$\mathbf{W} = (w_i)_{i=1}^n \in C([0, 1]; \mathbb{R}^n), \quad (8d)$$

$$\mathbf{\Theta} = (\theta_1, \dots, \theta_n) \in C([0, 1]; \mathbb{R}^{1 \times n}), \quad (8e)$$

$$\mathbf{Q} = (q_i)_{i=1}^n \in \mathbb{R}^n, \quad (8f)$$

with  $\lambda_1(x), \dots, \lambda_n(x), \mu(x) > 0$  for all  $x \in [0, 1]$ .

In order to compare the solution to (4), (5) with the solution to (6), (7), we transform the system (6), (7) from

$E$  to  $E_c$  by utilizing a linear transform  $\mathcal{F}_n$  that maps  $\mathbb{R}^n$  to  $L^2([0, 1]; \mathbb{R})$ . That is, for any  $\mathbf{b} = (b_i)_{i=1}^n \in \mathbb{R}^n$ , we define  $\mathcal{F}_n \mathbf{b} = \sum_{i=1}^n b_i \chi_{((i-1)/n, i/n]}$ , where  $\chi_{((i-1)/n, i/n]}$  denotes the indicator function of the interval  $(i-1)/n, i/n]$ . Now, applying the transform  $\mathcal{F} = \text{diag}(\mathcal{F}_n, 1)$  to (6), (7) from the left, we get

$$\mathcal{F}_n \mathbf{u}_t(t, x) + \mathcal{F}_n \mathbf{\Lambda}(x) \mathcal{F}_n^* \mathcal{F}_n \mathbf{u}_x(t, x) =$$

$$\frac{1}{n} \mathcal{F}_n \mathbf{\Sigma}(x) \mathcal{F}_n^* \mathcal{F}_n \mathbf{u}(t, x) + \mathcal{F}_n \mathbf{W}(x) v(t, x), \quad (9a)$$

$$v_t(t, x) - \mu(x) v_x(t, x) =$$

$$\frac{1}{n} \mathbf{\Theta}(x) \mathcal{F}_n^* \mathcal{F}_n \mathbf{u}(t, x), \quad (9b)$$

with boundary conditions

$$\mathcal{F}_n \mathbf{u}(t, 0) = \mathcal{F}_n \mathbf{Q} v(t, 0), \quad v(t, 1) = U(t), \quad (10)$$

where we used  $\mathcal{F}_n^* \mathcal{F}_n = I_n$  with  $\mathcal{F}_n^* g = \left( n \int_{(i-1)/n}^{i/n} g(\zeta) d\zeta \right)$

for any  $g \in L^2([0, 1]; \mathbb{R})$ . Now, introducing a new variable  $u^n = \mathcal{F}_n \mathbf{u}$  and step functions  $\sigma^n, \theta^n, \lambda^n, W^n, Q^n$  such that

$$\int_0^1 \sigma^n u^n d\zeta = \frac{1}{n} \mathcal{F}_n \mathbf{\Sigma} \mathcal{F}_n^* u^n, \quad \int_0^1 \theta^n u^n d\zeta = \frac{1}{n} \mathbf{\Theta} \mathcal{F}_n^* u^n, \quad (11a)$$

$$\lambda^n = \mathcal{F}_n \mathbf{\Lambda} \mathcal{F}_n^*, \quad W^n = \mathcal{F}_n \mathbf{W}, \quad Q^n = \mathcal{F}_n \mathbf{Q}, \quad (11b)$$

the transformed system (9), (10) can be written equivalently (on  $E_c$ ) as an  $\infty + 1$  hyperbolic system

$$u_t^n(t, x, y) + \lambda^n(x, y) u_x^n(t, x, y) = \int_0^1 \sigma^n(x, y, \zeta) u^n(t, x, \zeta) d\zeta + W^n(x, y) v(t, x), \quad (12a)$$

$$v_t(t, x) - \mu(x) v_x(t, x) =$$

$$\int_0^1 \theta^n(x, \zeta) u^n(t, x, \zeta) d\zeta, \quad (12b)$$

with boundary conditions

$$u^n(t, 0, y) = Q^n(y) v(t, 0), \quad v(t, 1) = U(t). \quad (13)$$

As the solutions  $(u, v)$  and  $(u^n, v)$  of the two systems (4), (5) and (12), (13), respectively, depend continuously on the respective parameters, we call (4), (5) a continuum approximation of (12), (13) if the parameters of the former are chosen such that, for a given (sufficiently small)  $\varepsilon > 0$ , they satisfy

$$\max_{x \in [0, 1]} \|\lambda^n(x, \cdot) - \lambda(x, \cdot)\|_{L^2([0, 1]; \mathbb{R})} + \max_{x \in [0, 1]} \|\lambda_x^n(x, \cdot) - \lambda_x(x, \cdot)\|_{L^2([0, 1]; \mathbb{R})} < \varepsilon, \quad (14a)$$

$$\max_{x \in [0, 1]} \|\sigma^n(x, \cdot) - \sigma(x, \cdot)\|_{L^2([0, 1]^2; \mathbb{R})} < \varepsilon, \quad (14b)$$

$$\max_{x \in [0, 1]} \|W^n(x, \cdot) - W(x, \cdot)\|_{L^2([0, 1]; \mathbb{R})} < \varepsilon, \quad (14c)$$

$$\max_{x \in [0, 1]} \|\theta^n(x, \cdot) - \theta(x, \cdot)\|_{L^2([0, 1]; \mathbb{R})} < \varepsilon, \quad (14d)$$

$$\|Q^n - Q\|_{L^2([0, 1]; \mathbb{R})} < \varepsilon. \quad (14e)$$

The approximation property of the parameter as per (14) is enough for designing approximate state-feedback laws for large-scale  $n + 1$  systems, as discussed in the next section. For considering the proximity of solutions to

(4), (5) and (12), (13), one additionally needs to establish proximity of the initial conditions such that  $\|u_0 - \mathcal{F}\mathbf{u}_0\|_{L^2([0,1];L^2([0,1];\mathbb{R}))}$  is sufficiently small. Under these conditions (and identical inputs in (5), (13)), we have shown that the solutions to (4), (5) and (12), (13) stay (relatively) close to each other on compact time intervals (Humaloja and Bekiaris-Liberis, 2025c, Thm 6.1). Moreover, this approximation property has been utilized in continuum observer-based output-feedback design in Humaloja and Bekiaris-Liberis (2025a).

### 3. APPLICATION TO STABILIZATION VIA APPROXIMATION OF BACKSTEPPING KERNELS

A stabilizing backstepping state-feedback control law for  $n + 1$  systems (6), (7) can be constructed as

$$U(t) = \int_0^1 \left( \frac{1}{n} \mathbf{K}(1, \xi) \mathbf{u}(t, \xi) + L(1, \xi) v(t, \xi) \right) d\xi, \quad (15)$$

where  $\mathbf{K} = [K_1, \dots, K_n]$  and  $L$  satisfy the kernel equations

$$\mu(x) \mathbf{K}_x(x, \xi) - \mathbf{K}_\xi(x, \xi) \mathbf{\Lambda}(\xi) - \mathbf{K}(x, \xi) \mathbf{\Lambda}'(\xi) = \frac{1}{n} \mathbf{K}(x, \xi) \mathbf{\Sigma}(\xi) + L(x, \xi) \mathbf{\Theta}(\xi), \quad (16a)$$

$$\mu(x) L_x(x, \xi) + L_\xi(x, \xi) \mu(\xi) + L(x, \xi) \mu'(\xi) = \frac{1}{n} \mathbf{K}(x, \xi) \mathbf{W}(\xi), \quad (16b)$$

on  $\mathcal{T}$ , with boundary conditions

$$-\mathbf{\Theta}(x) = \mathbf{K}(x, x) \mathbf{\Lambda}(x) + \mu(x) \mathbf{K}(x, x), \quad (17a)$$

$$L(x, 0) = \frac{1}{\mu(0)} \frac{1}{n} \mathbf{K}(x, 0) \mathbf{\Lambda}(0) \mathbf{Q}. \quad (17b)$$

One could proceed on the basis of Section 2 and seek for a continuum counterpart for the kernel equations by applying  $\mathcal{F}^*$  to (16), (17) from the right<sup>1</sup> and using  $\mathcal{F}_n^* \mathcal{F}_n = I_n$ . Another approach would be to derive a stabilizing backstepping control law for  $\infty + 1$  systems directly. Both approaches lead to continuum kernel equations

$$\mu(x) K_x(x, \xi, y) - K_\xi(x, \xi, y) \lambda(\xi, y) - K(x, \xi, y) \lambda_\xi(\xi, y) = \int_0^1 K(x, \xi, \zeta) \sigma(\xi, \zeta, y) d\zeta + L(x, \xi) \theta(\xi, y), \quad (18a)$$

$$\mu(x) L_x(x, \xi) + L_\xi(x, \xi) \mu(\xi) + L(x, \xi) \mu'(\xi) = \int_0^1 K(x, \xi, \zeta) W(\xi, \zeta) d\zeta, \quad (18b)$$

on  $\mathcal{T} \times [0, 1]$ , with boundary conditions

$$-\theta(x, y) = K(x, x, y) \lambda(x, y) + \mu(x) K(x, x, y), \quad (19a)$$

$$L(x, 0) = \frac{1}{\mu(0)} \int_0^1 K(x, 0, \zeta) \lambda(0, \zeta) Q(\zeta) d\zeta, \quad (19b)$$

and the respective stabilizing control law for (4), (5) is

<sup>1</sup> Or transposing (16), (17) and applying  $\mathcal{F}$  from the left, which is the approach taken in Humaloja and Bekiaris-Liberis (2025c).

$$U(t) = \int_0^1 \left( \int_0^1 K(1, \xi, y) u(t, \xi, y) dy + L(1, \xi) v(t, \xi) \right) d\xi. \quad (20)$$

Writing the first term in the control law (15) in terms of  $u^n = \mathcal{F}_n \mathbf{u}$ , there is a step function  $K^n$  such that

$$\int_0^1 K^n(1, \xi, y) u^n(t, \xi, y) dy = \frac{1}{n} \mathbf{K}(1, \xi) \mathcal{F}_n^* u^n. \quad (21)$$

Conversely, if one inserts  $(u^n, v)$  to (20), the first term becomes

$$\int_0^1 K(1, \xi, y) u^n(t, \xi, y) dy = \frac{1}{n} \tilde{\mathbf{K}}(1, \xi) \mathbf{u}(t, \xi), \quad (22)$$

where  $\tilde{\mathbf{K}} = [\tilde{K}_1, \dots, \tilde{K}_n]$  with

$$\tilde{K}_i(1, \xi) = n \int_{(i-1)/n}^{i/n} K(1, \xi, y) dy. \quad (23)$$

It has been shown in (Humaloja and Bekiaris-Liberis, 2025c, Lem. 4.3) that the difference of  $(K^n, L)$  and  $(K, L)$ , as well as that of  $(\mathbf{K}, L)$  and  $(\tilde{\mathbf{K}}, L)$ , tends to zero as  $\varepsilon \rightarrow 0$  in (14). Consequently, by (Humaloja and Bekiaris-Liberis, 2025c, Thm 4.1), the approximate control law

$$U(t) = \int_0^1 \left( \frac{1}{n} \tilde{\mathbf{K}}(1, \xi) \mathbf{u}(t, \xi) + L(1, \xi) v(t, \xi) \right) d\xi, \quad (24)$$

exponentially stabilizes the  $n + 1$  system (6), (7), provided that  $\varepsilon$  in (14) is sufficiently small. The potential benefit of the approximate control law (24) is that, instead of having to solve  $n + 1$  exact kernels from (16), (17), it is enough to solve 2 kernels from (18), (19). While comparing the complexity of solving (16), (17) with solving (18), (19) is not straightforward due to (18), (19) involving a 3-D kernel  $K$ , the power series method that was employed to solve (18), (19) in Humaloja and Bekiaris-Liberis (2025b) shows potential in reducing computational complexity. In any case, the continuum kernel equations (18), (19) are independent of  $n$ , and hence, inevitably easier to handle computationally than (16), (17) when  $n$  becomes sufficiently large.

### 4. DISCUSSION AND OPEN PROBLEMS

We have demonstrated that continuum hyperbolic PDEs can serve as continuum approximations of large-scale hyperbolic PDEs. Technically, the large-scale aspect of the approximated hyperbolic PDE is not a requirement, but the computational benefits of continuum approximations become more apparent for large-scale systems. The key property of continuum approximations is that the parameters of the constructed continuum system (4), (5) are chosen such that they approximate the respective parameters of the  $n + 1$  system (6), (7), i.e., that  $\varepsilon$  in (14) is sufficiently small. Hence, construction of a continuum approximation (4), (5) requires a mechanism to construct functions on the basis of (14), based on given parameters of an  $n + 1$  system and a given  $\varepsilon > 0$ .

Considering the continuum parameter  $Q$  as an example, we represent the function as

$$Q(y) = \sum_{k=0}^N c_k f_k(y), \quad (25)$$

for some coefficients  $c_k$  and functions  $f_k$  for  $k = 0, \dots, N$ . In (Humaloja and Bekiaris-Liberis, 2025b, Sect. 6), a simple choice of  $f_k(y) = y^k$  was employed in combination with least-squares fit for finding the coefficients  $c_k$  based on the  $q_i$  data, which was directly compatible with the power series method for solving the continuum kernel equations (18), (19). However, the functions  $f_k$  in (25) can be chosen in countless ways, and it remains an open problem as to which choice would be, in some sense, optimal for a particular continuum approximation. In particular, employing neural networks in constructing continuum approximations and solving the respective kernel equations is an interesting topic of future research. On a related note, in addition to the accuracy as per (14), the constructed continuum approximation should be, in some sense, simple, so that it is computationally beneficial over the original large-scale system.

Another aspect that may affect the continuum approximations is the interpretation of a large-scale system as a continuum system. In Section 2, we introduced the transform  $\mathcal{F}_n$ , transforming vectors in  $\mathbb{R}^n$  to step functions in  $L^2([0, 1]; \mathbb{R})$ , which resulted in the approximation condition (14). However, the choice of the transform  $\mathcal{F}_n$  is by no means unique, nor is the choice of the continuum space  $E_c$ , which could be alternatively based on another function space such as  $L^\infty$ , or a sequence space, e.g.,  $\ell^2$  or  $\ell^\infty$ . Such alternative frameworks for the continuum system or transforms alternative to  $\mathcal{F}_n$  have thus far been unexplored in the existing literature, as well as the types of approximation conditions that they may give rise to.

## REFERENCES

- Alleaume, V. and Krstic, M. (2025). Ensembles of hyperbolic PDEs: Stabilization by backstepping. *IEEE Trans. Automat. Control*, 70, 905–920.
- Auriol, J. and Bresch-Pietri, D. (2022). Robust state-feedback stabilization of an underactuated network of interconnected  $n + m$  hyperbolic PDE systems. *Automatica*, 136, 110040.
- Auriol, J. and Di Meglio, F. (2016). Minimum time control of heterodirectional linear coupled hyperbolic PDEs. *Automatica*, 71, 300–307.
- Bikia, V. (2021). *Non-invasive monitoring of key hemodynamical and cardiac parameters using physics-based modelling and artificial intelligence*. Ph.D. thesis, EPFL.
- Burkhardt, M., Yu, H., and Krstic, M. (2021). Stop-and-go suppression in two-class congested traffic. *Automatica*, 123, 109381.
- Coron, J.M., Hu, L., and Olive, G. (2017). Finite-time boundary stabilization of general linear hyperbolic balance laws via Fredholm backstepping transformation. *Automatica*, 84, 95–100.
- Coron, J.M., Hu, L., Olive, G., and Shang, P. (2021). Boundary stabilization in finite time of one-dimensional linear hyperbolic balance laws with coefficients depending on time and space. *J. Differ. Equ.*, 271, 1109–1170.
- Di Meglio, F., Vazquez, R., and Krstic, M. (2013). Stabilization of a system of  $n + 1$  coupled first-order hyperbolic linear PDEs with a single boundary input. *IEEE Trans. Automat. Control*, 58, 3097–3111.
- Guan, L., Prieur, C., Zhang, L., Prieur, C., Georges, D., and Bellemain, P. (2020). Transport effect of COVID-19 pandemic in France. *Annu. Rev. Control*, 50, 394–408.
- Herty, M. and Klar, A. (2003). Modeling, simulation, and optimization of traffic flow networks. *SIAM J. Sci. Comput.*, 25, 1066–1087.
- Hu, L., Di Meglio, F., Vazquez, R., and Krstic, M. (2016). Control of homodirectional and general heterodirectional linear coupled hyperbolic PDEs. *IEEE Trans. Automat. Control*, 61, 3301–3314.
- Humaloja, J.P. and Bekiaris-Liberis, N. (2025a). Continuum-based output-feedback stabilization of large-scale  $n + m$  hyperbolic PDEs. In *IEEE Conf. Decis. Control*, 121–126.
- Humaloja, J.P. and Bekiaris-Liberis, N. (2025b). On computation of approximate solutions to large-scale backstepping kernel equations via continuum approximation. *Syst. Control Lett.*, 196, 105982.
- Humaloja, J.P. and Bekiaris-Liberis, N. (2025c). Stabilization of a class of large-scale systems of linear hyperbolic PDEs via continuum approximation of exact backstepping kernels. *IEEE Trans. Automat. Control*, 70, 5957–5972.
- Humaloja, J.P. and Bekiaris-Liberis, N. (2026a). Backstepping control of continua of linear hyperbolic PDEs and application to stabilization of large-scale  $n + m$  coupled hyperbolic PDE systems. *Automatica*, 183, 112647.
- Humaloja, J.P. and Bekiaris-Liberis, N. (2026b). Backstepping stabilization of  $\infty + \infty$  continua of linear hyperbolic systems. In *American Control Conference*.
- Kitsos, C., Besancon, G., and Prieur, C. (2022). High-gain observer design for a class of quasi-linear integro-differential hyperbolic systems-application to an epidemic model. *IEEE Trans. Automat. Control*, 67, 292–303.
- Lin, X., Vazquez, R., and Krstic, M. (2024). Towards a MATLAB Toolbox to compute backstepping kernels using the power series method. In *IEEE Conf. Decis. Control*, 446–451.
- Malekzadeh, M., Papamichail, I., and Papageorgiou, M. (2021). Linear-quadratic regulators for internal boundary control of lane-free automated vehicle traffic. *Control Eng. Pract.*, 115, 104912.
- Reymond, P., Merenda, F., Perren, F., Rufenacht, D., and Stergiopoulos, N. (2009). Validation of a one-dimensional model of the systemic arterial tree. *Am. J. Physiol. Heart Circ. Physiol.*, 297, H208–H222.
- Vazquez, R., Chen, G., Qiao, J., and Krstic, M. (2023a). The power series method to compute backstepping kernel gains: theory and practice. In *IEEE Conf. Decis. Control*, 8162–8169.
- Vazquez, R., Zhang, J., Qi, J., and Krstic, M. (2023b). Kernel well-posedness and computation by power series in backstepping output feedback for radially-dependent reaction-diffusion PDEs on multidimensional balls. *Syst. Control Lett.*, 177, 105538.
- Yu, H. and Krstic, M. (2021). Output feedback control of two-lane traffic congestion. *Automatica*, 125, 109379.